

# Applying Coordinate Transformations to Multi-DOF Shaker Control

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The dynamic characteristics of test fixtures, test articles and multiple shakers or actuators for multi-degree-of-freedom (MDOF) dynamic testing may cause the number of desired degrees of freedom for MDOF operations to not match the number of exciters or control transducers being used. Thus, it is often necessary to transform the response from multiple control transducers and the capability of multiple exciters that are being used from actuator space to MDOF space and vice-versa to effectively perform such tests. These operations are now capable of being directly performed within the software used for MDOF control to increase test flexibility and greatly reduce test setup time. This article describes current applications of these techniques with multiple shakers and examines the control of torsional motion in large plates as a by-product of coordinate transformation.

Historically, both single-axis and multiple-axis testing of large structures using multiple exciters has relied on using large fixtures, or tables, for attachment and positioning of the test article. Typically, the tables are designed using Finite Element Analysis (FEA). Often, cursory acceptance tests of the final design and fabrication are run, using single or multiple exciters to determine the modal characteristics of the tables. Despite these efforts and precautions, many cases exist in which important deformation frequencies and characteristics, which can fall into the operating frequency range of the loaded tables, go undetected until tests are actually being run.

The use of large tables and multiple exciters for multiple-axis testing has been most prevalent in the seismic testing community. In these cases, 6-DOF testing is typical. Testing parameters are specified in the three translational axes, X, Y, and Z. Profiles for roll, pitch, and yaw may also be specified or designated to be "kept to a minimum." For many seismic applications, six, seven or eight hydraulic actuators will be attached to the test table, and it will be necessary to transform these actuators and their control points into a series of control and drive parameters suitable for controlling X, Y, Z, roll, pitch, and yaw – the classic six-rigid-body DOF parameters. This coordinate transformation, which traditionally has been done in hardware by summing or subtracting control signals can now be done much more efficiently in software, optimally right by the digital vibration control system.

## Some practical considerations

To keep track of actual control measurements as opposed to 'created' control vectors, and actual actuator drive signals as opposed to drive vectors, a consistent terminology and numbering scheme must be adopted. Since there will be both an input transformation from actual control accelerometers and/or control LVDTs (Linear Variable Differential Transformers) to (typically) a 6-DOF definition of control vectors and an output transformation from (typically) the 6-DOF drive vectors to the actual number of actuator drives, we should adopt an unambiguous means to refer to each parameter. In this article:

*I* = The actual control transducers that appear in a channel table and that require a selected coupling or signal conditioning will be referred to as *control inputs*.

*C* = The vector or matrix that results from transforming the control inputs (e.g. eight accelerometers to 6 DOF) will be referred to as *controls*.

*D* = The drive vector or matrix that results from the control calculations will be referred to as *drives*.

*O* = The signals that result from applying the output transformation to the drives will be referred to as *drive outputs* and are used to drive each actuator or shaker.

Figure 1 shows a 40-ton seismic table installed in China for seismic simulation. Eight servohydraulic actuators drive the table with four vertical and two each (opposed) in the horizontal and lateral directions. Controlled operation is from 0.1 Hz to 100 Hz. with maximum displacements of greater than 200 mm p-p. Loads of up to 60 tons can be accommodated.

## Mapping the Input Signal Transformations

In this installation, eight LVDTs are used for "inner loop" position control, and eight compensated accelerometers mounted at the table edges are used for "outer loop" dynamic control. Since the primary purpose of most tests performed with this system is to simulate 6-DOF motion, a transformation from eight control accelerometers to 6 DOF is required. For this example the *input transformation matrix* appears as:

$$\begin{Bmatrix} X \\ Y \\ R_Z \\ Z \\ R_Y \\ R_X \end{Bmatrix} = \begin{bmatrix} 0.5 & -0.5 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -0.5 & 0.5 & 0 & 0 & 0 & 0 \\ -0.41 & -0.41 & -0.41 & -0.41 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.25 & 0.25 & 0.25 & 0.25 \\ 0 & 0 & 0 & 0 & -0.41 & -0.41 & 0.41 & 0.41 \\ 0 & 0 & 0 & 0 & 0.41 & -0.41 & -0.41 & 0.41 \end{bmatrix} \begin{Bmatrix} X_1 \\ X_2 \\ Y_1 \\ Y_2 \\ Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \end{Bmatrix} \quad (1)$$

Since the X and Y actuators are both in opposing pairs, the signs on one of them must be changed to maintain a right-handed coordinate system. In this example, each of the vertical actuators carries equal weight. The rotational components are calculated by appropriate summation of transducer signals to create yaw, roll, and pitch values and also by taking their radius of separation into account.

Even in a 40-ton seismic table, a heavy load can force unwanted table responses down into the operating frequency range. In the case described above, because of just such an occurrence, a 9th vertical accelerometer was added at the center of the table. To give as much sensitivity as possible to center motion, the 9th accelerometer was given 60% weighting. Thus a *new input transformation matrix* was developed as follows:

$$\begin{Bmatrix} X \\ Y \\ R_Z \\ Z \\ R_Y \\ R_X \end{Bmatrix} = \begin{bmatrix} 0.5 & -0.5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -0.5 & 0.5 & 0 & 0 & 0 & 0 & 0 \\ -0.41 & -0.41 & -0.41 & -0.41 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.1 & 0.1 & 0.1 & 0.1 & 0.6 \\ 0 & 0 & 0 & 0 & -0.41 & -0.41 & 0.41 & 0.41 & 0 \\ 0 & 0 & 0 & 0 & 0.41 & -0.41 & -0.41 & 0.41 & 0 \end{bmatrix} \begin{Bmatrix} X_1 \\ X_2 \\ Y_1 \\ Y_2 \\ Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \end{Bmatrix} \quad (2)$$

In this transformation, X, Y and the rotational component transformations were unchanged but the four-corner Z accelerometers were reduced in 'participation' in favor of a more sensitive measure of motion at the center of the table. Combined with



Figure 1. 6 meter x 6 meter seismic table with eight actuators – 4 Z, 2 X, 2 Y.

other electronic modifications, this was enough to greatly improve controlled table responses under a wide range of test articles and loads. However, this transformation did not use this added information with its full geometric import and thus did not resolve the problems with torsion that were behind most of the performance problems that were encountered. This topic is discussed later.

### Mapping the Output Drive Transformations

In the 6 m x 6 m table installation, the actuators are located at the four corners, with four horizontal and four vertical actuators. A part of this is seen in Figure 2.

The drive output transformation must map the 6-DOF drives to the eight actual servo-valve drive signals as follows:

$$\begin{Bmatrix} D_{x_1} \\ D_{x_2} \\ D_{y_1} \\ D_{y_2} \\ D_{z_1} \\ D_{z_2} \\ D_{z_3} \\ D_{z_4} \end{Bmatrix} = \begin{bmatrix} 1.0 & 0 & -0.93 & 0 & 0 & 0 & 0 & 0 \\ -1.0 & 0 & -0.93 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1.0 & -0.93 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1.0 & -0.93 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1.0 & -0.92 & 0.93 & 0 & 0 \\ 0 & 0 & 0 & 1.0 & -0.92 & -0.93 & 0 & 0 \\ 0 & 0 & 0 & 1.0 & 0.92 & 0.93 & 0 & 0 \\ 0 & 0 & 0 & 1.0 & 0.92 & -0.93 & 0 & 0 \end{bmatrix} \begin{Bmatrix} D_X \\ D_Y \\ D_{R_z} \\ D_Z \\ D_{R_y} \\ D_{R_x} \end{Bmatrix} \quad (3)$$

Note that the drive output transformation is heavily influenced by the geometry of the table design and actuator placements. Regardless of how many control transducers are used, as in Equations 1 and 2, in this example there are always eight actuators associated with 6 DOF, and Equation 3 remains valid. However, if we consider deformational degrees of freedom in addition to the rigid body degrees of freedom that we've so far discussed, a more comprehensive transformation matrix needs to be considered.

### Taking Torsional Responses into consideration

In general, many input and output transformations are possible. For example, if we suspect that large torsional responses may be excited by sizeable payloads, then we may want to consider adding torsional control elements to our transformation matrix. Torsion is an additional independent deformational degree of freedom if table deformation is occurring. If we consider horizontal and vertical torsion, which can be represented by  $T_H$  and  $T_V$ , then we can formulate a 6-DOF plus two torsional DOF input transformation as:



Figure 2. Corner view showing attachment of one horizontal and one lateral actuator.

$$\begin{Bmatrix} X \\ Y \\ R_z \\ T_H \\ Z \\ R_y \\ R_x \\ T_V \end{Bmatrix} = \begin{bmatrix} 0.5 & -0.5 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -0.5 & 0.5 & 0 & 0 & 0 & 0 \\ -0.41 & -0.41 & -0.41 & -0.41 & 0 & 0 & 0 & 0 \\ 0.5 & 0.5 & -0.5 & -0.5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.25 & 0.25 & 0.25 & 0.25 \\ 0 & 0 & 0 & 0 & -0.41 & -0.41 & 0.41 & 0.41 \\ 0 & 0 & 0 & 0 & 0.41 & -0.41 & -0.41 & 0.41 \\ 0 & 0 & 0 & 0 & 0.5 & -0.5 & 0.5 & -0.5 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ y_1 \\ y_2 \\ z_1 \\ z_2 \\ z_3 \\ z_4 \end{Bmatrix} \quad (4)$$

This considers four vertical accelerometers without the middle accelerometer. The corresponding drive output (with torsion drives) transformation becomes:

$$\begin{Bmatrix} D_{x_1} \\ D_{x_2} \\ D_{y_1} \\ D_{y_2} \\ D_{z_1} \\ D_{z_2} \\ D_{z_3} \\ D_{z_4} \end{Bmatrix} = \begin{bmatrix} 1 & 0 & -0.93 & 0.93 & 0 & 0 & 0 & 0 \\ -1 & 0 & -0.93 & 0.93 & 0 & 0 & 0 & 0 \\ 0 & -1 & -0.93 & -0.93 & 0 & 0 & 0 & 0 \\ 0 & 1 & -0.93 & -0.93 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -0.92 & 0.93 & 0.5 \\ 0 & 0 & 0 & 0 & 1 & -0.92 & -0.93 & -0.5 \\ 0 & 0 & 0 & 0 & 1 & 0.92 & -0.93 & 0.5 \\ 0 & 0 & 0 & 0 & 1 & 0.92 & 0.93 & -0.5 \end{bmatrix} \begin{Bmatrix} D_X \\ D_Y \\ R_z \\ T_H \\ Z \\ R_y \\ R_x \\ T_V \end{Bmatrix} \quad (5)$$

By using powerful coordinate transformations, we can create an 8-DOF control strategy from our eight or nine control accelerometers in this example. Even though only six independent rigid body DOF are possible, additional deformational DOF exist. Thus, table deformation creates two additional torsional DOF that are independent of the first six DOF, which are observable and controllable given the current instrumentation and actuator configuration of our example.

### Application Examples

An aluminum plate, 16.5 x 20.5 in and 5/16 in. thick, was attached via stingers to four 50-lb electrodynamic shakers (see Figure 3). Several tests, all using multiple-input, multiple-output (MIMO) control techniques were conducted. In all cases, four vertical (Z+) accelerometers were used for measurement and control.

**Direct Actuator Control.** In the first example, standard "square-control" MIMO technology was applied as shown in Figure 4, using four vertical actuators (shakers) and four control accelerometers. A control frequency range from 5 to 200 Hz was selected, and MIMO random excitation and control was used. Although comparable MIMO sine results could have also been discussed, to limit the breadth of the article, only the MIMO random results are discussed here. Each of the four con-

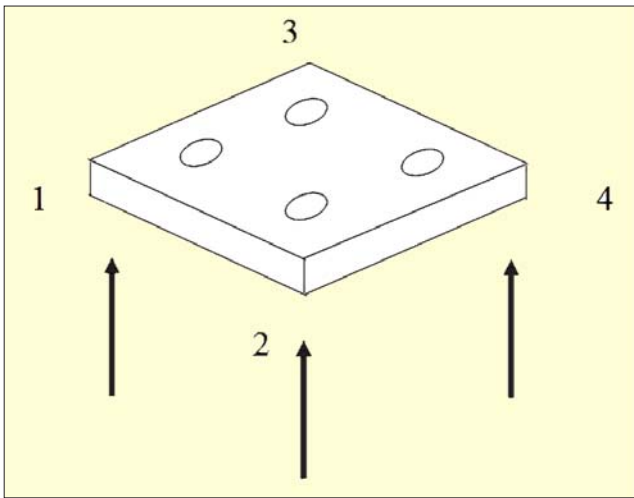


Figure 3. Square aluminum plate driven by four small vertical shakers.

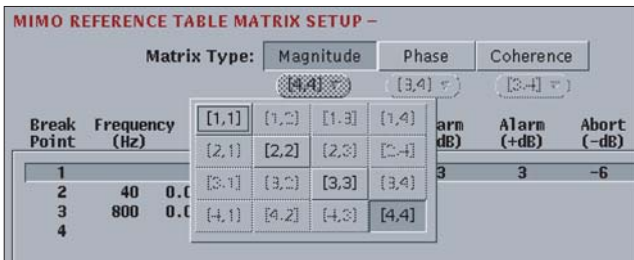


Figure 4. Typical  $4 \times 4$  matrix for specifying control PSDs along major diagonal.

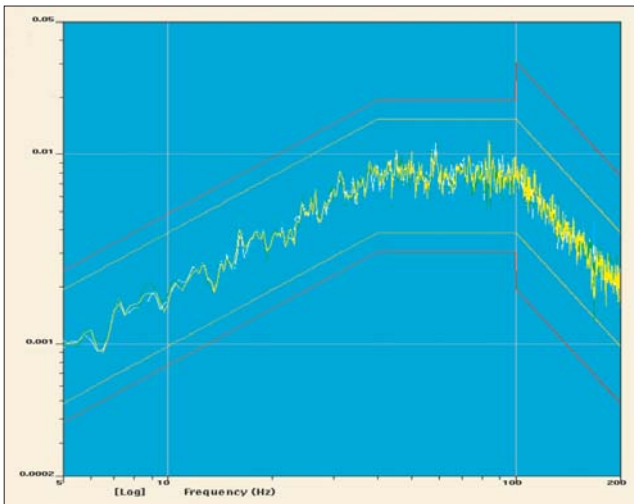


Figure 5. Four control accelerometers for direct 'square' MIMO control.

control PSDs (Power spectral Densities) was specified to be identical. The control phase was set for  $0^\circ$ , and the control coherence was specified as 0.9. If the coherence is set to one between them, then a singular Spectral Density Matrix (SDM) results.

Figure 5 shows typical control signals for the four vertical accelerometers mounted near the attachment points of the plate/shakers. The four PSD traces nearly overlay each other, especially at the lower frequencies. These results were for a designated coherence of 0.9. Had a higher coherence been used (0.95 or 0.99, for example), even higher similarity between control PSDs could result. However, direct actuator control with higher relative coherence is difficult.

Although the magnitude, PSD, and controls shown in Figure 5 are quite good for this plate, the control phase and coherence seen in Figure 6 and the following figures for impedances and drives, indicate that several severe antiresonances are present that suggest a more optimal control strategy.

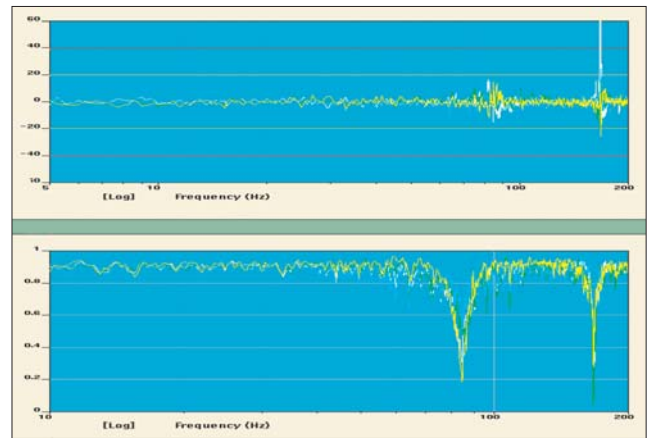


Figure 6. Control phase and coherence for direct square control.

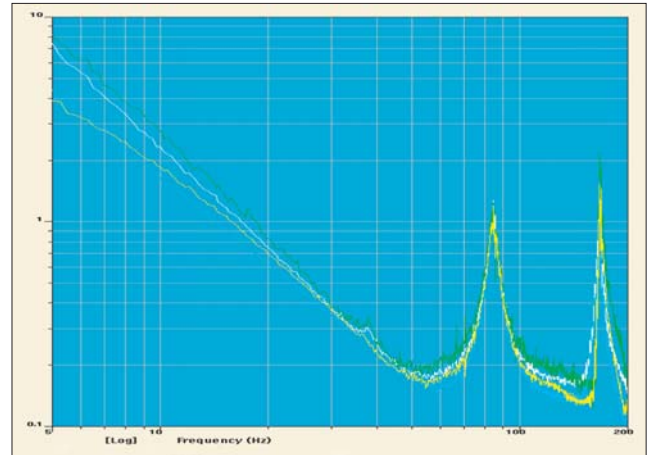


Figure 7. Driving point impedances at four shakers.

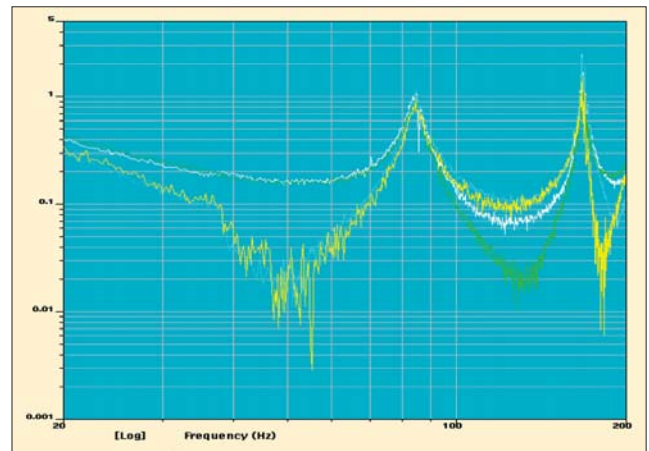


Figure 8. Some off-diagonal Impedances from the  $4 \times 4$  impedance matrix.

Figure 7 shows the impedances from along the major diagonal of the  $4 \times 4$  impedance matrix. At the lower frequencies, they exhibit similarity in pairs, with shakers 1 and 2 showing close similarity to one another, as do shakers 3 and 4.

In Figure 8, again there are similarities between pairs of impedance functions below the first-plate dynamic mode of 84 Hz. Depending on how the phase of these impedance component functions differ, this could be an indication of a twisting or torsional response in the plate and the attempt by the control system to compensate for it.

Figure 9 shows that the drive signals are well matched and driving quite hard to overcome antiresonant responses at 84 Hz and 170 Hz. Antiresonances such as these typically limit the frequency range over which effective multi-actuator control can be accomplished. So, overcoming these antiresonances

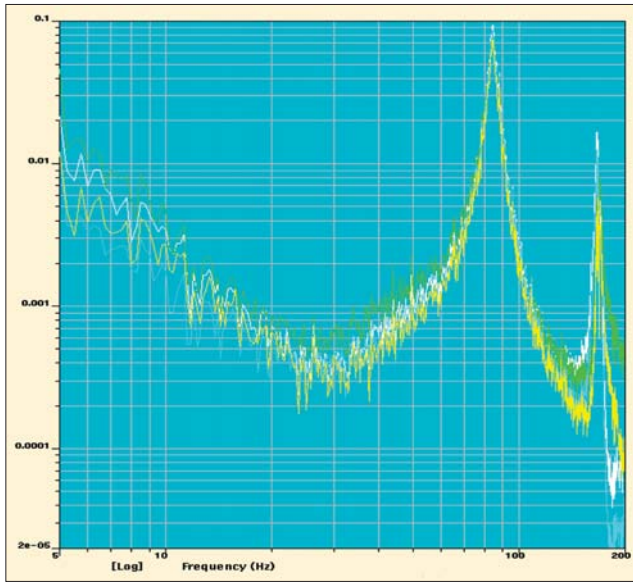


Figure 9. Four drive signals to achieve control (Figure 5).

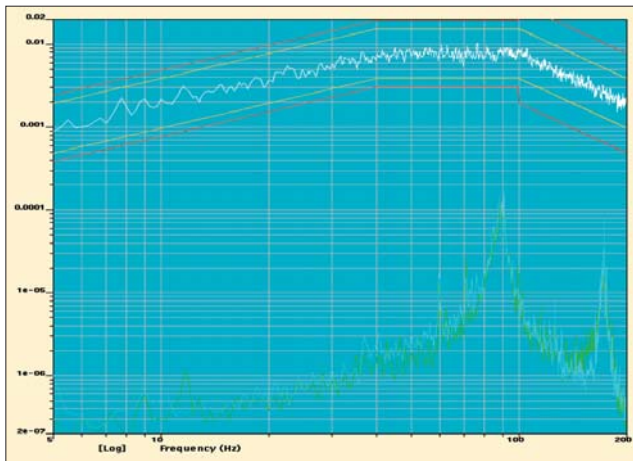


Figure 10. Control of vertical (white) and suppression of roll (green) and pitch (blue) motion; 3-DOF control.

is an important part of multi-actuator control and multi-actuator fixture and table design. From the control side, I/O transformations are effective in this regard, as will be shown here later.

**3-DOF Control.** In the second example, the same four shakers and (vertical) control accelerometers were used. In this case, however, a transformation matrix was used to create “virtual controls” for roll-and-pitch motion. With four vertical accelerometers, therefore, a control condition was established to control: vertical translational motion, roll rotational motion, and pitch rotational motion.

Also, for this test, the phase between the three control vectors was set to  $0^\circ$ , and the coherence was set to 0. (By setting the coherence to 0, the excitations are essentially uncoupled and the definition of phase becomes essentially immaterial). Also, the reference PSD for roll and pitch were set to be the same but at a much lower level than the reference PSD for the vertical direction.

The following coordinate transformation matrix was established to create the desired control vectors:

$$\begin{Bmatrix} z \\ r \\ p \end{Bmatrix} = \begin{bmatrix} 0.25 & 0.25 & 0.25 & 0.25 \\ 0.25 & 0.25 & -0.25 & -0.25 \\ 0.25 & -0.25 & 0.25 & -0.25 \end{bmatrix} \begin{Bmatrix} A1 \\ A2 \\ A3 \\ A4 \end{Bmatrix} \quad (6)$$

In this example, the four accelerometers in various phase combinations are used to control roll and pitch. The main goal of this test is to control the vertical response as closely as pos-

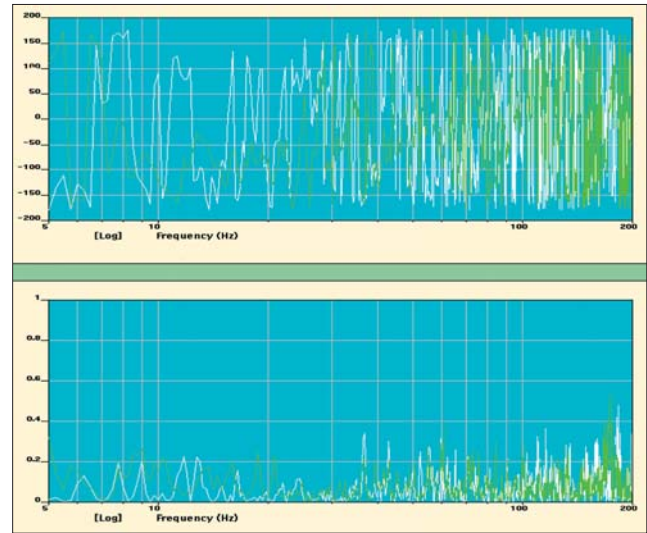


Figure 11. Control phase and coherence for 3-DOF case.

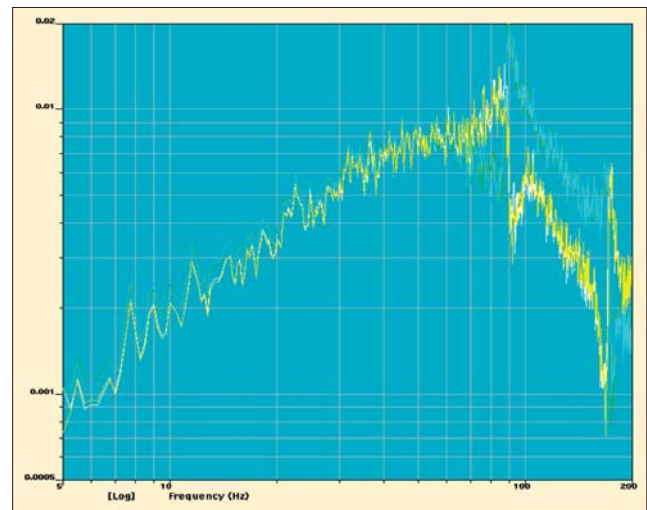


Figure 12. Four vertical accelerometer inputs during 3-DOF control.

sible to a described reference PSD and to minimize rotational responses, as is shown in the following Figures 10 and 11. Figure 10 shows the response of the translational degree of freedom Z and also the response roll and pitch degrees of freedom. The rotational responses are suppressed compared to the translational response. Figure 11 shows that the control phase is essentially zero and random at all frequencies and that the coherence is essentially zero at all frequencies.

The four vertical accelerometer PSDs measured directly from the table during 3-DOF control are shown in Figure 12. Compare this with the vertical control vector [1,1] in Figure 10. It is obvious that, by using the 3-DOF transformation matrix to control vertical motion and reduce rotational motion, we actually achieve an average vertical control. This is similar to the result that one achieves in single-exciter testing when one controls the average response. However, here the result is better because there is an obvious suppression of the roll and pitch motions shown in Figure 10.

Figure 13 shows a very interesting result. Coherence was achieved at all frequencies, as seen in Figure 11, even though the coherence between the vertical, roll, and pitch vectors was set to zero. The relationships between the four vertical accelerometers is much more sympathetic. The coherence between inputs is above 0.99 throughout most of the test range. The Phase between accelerometers 1 and 4 is close to zero throughout the entire test. But the phase of [1,2] and [1,3] goes through  $90^\circ$  near 90 Hz, indicating that a potentially severe twisting motion is taking place on the plate. This result is not totally surprising, since in this case, there is no conscious attempt to

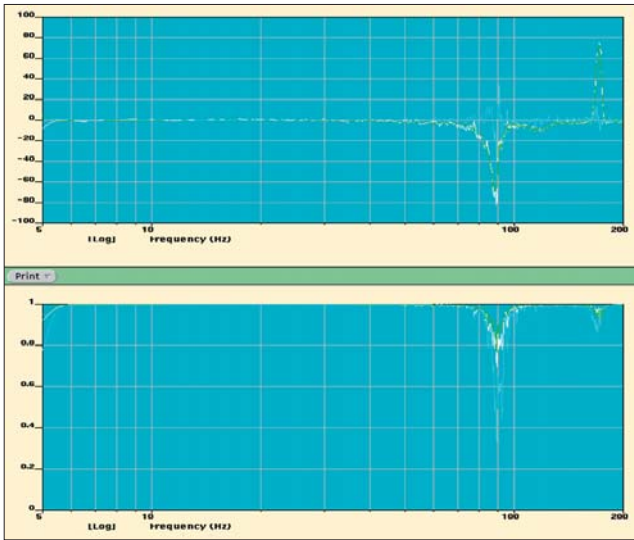


Figure 13. Input phase and coherence between vertical accelerometers, 3-DOF case.

control the torsional response of the plate.

This fosters the idea that perhaps a more uniform motion could be achieved on the plate if some attention was paid to potential plate torsional responses. This leads to example 3.

**4-DOF Control.** In the third example, the same four shakers and (vertical) control accelerometers were used. In this case, however, a transformation matrix was used to create “virtual controls” for roll, pitch, and torsional motion. So, with four vertical accelerometers, a control condition was established to control: *vertical* translational motion, *roll* rotational motion, *pitch* rotational motion, and *torsional* motion. Also for this test, the control phase was set to 0°, and the control coherence was set to 0. Also, the reference PSDs for roll, pitch, and torsion were set to be the same but at a much lower level than the reference PSD for the vertical direction. The primary goal here is to achieve excellent vertical control while minimizing roll, pitch, and torsional motion. The input transformation used to achieve this control condition is:

$$\begin{Bmatrix} z \\ r \\ p \\ t \end{Bmatrix} = \begin{bmatrix} 0.25 & 0.25 & 0.25 & 0.25 \\ 0.25 & 0.25 & -0.25 & -0.25 \\ 0.25 & -0.25 & 0.25 & -0.25 \\ 0.25 & -0.25 & -0.25 & 0.25 \end{bmatrix} = \begin{Bmatrix} A1 \\ A2 \\ A3 \\ A4 \end{Bmatrix} \quad (7)$$

By including provisions for controlling the torsional response in the control scheme, several benefits occur from a control standpoint. As shown in Figure 14, the vertical control response is extremely accurate with absolutely minimum error with respect to its desired reference spectrum. Also, each of the remaining rotational control responses, shown in rad/sec<sup>2</sup>, have been reduced to greater than 20 dB below the vertical value and show an improvement of 6 dB when compared with 3-DOF control. But by reducing the unwanted rotational responses significantly, other control improvements are achieved, as seen in Figures 15 and 16.

The control vector phase and coherence for the 4-DOF control case is shown in Figure 15. If compared with Figure 11, it can be seen that the ‘simple’ act of reducing torsional motion has a significant improvement in reducing the control coherence across the entire frequency range and at the same time maintains uncorrelated phase between all control vectors, as requested by the references that were entered.

The actual response of the four vertical accelerometers during full level 4-DOF control is seen in Figure 16. The four-accelerator PSD traces virtually overlay each other at all frequencies. Also note that the RMS level of each accelerometer is controlled to within ±0.4% of each other. Clearly, reducing the torsional response of the plate can offer a significant improvement in translational control; that is, achieving almost pure vertical motion in the table’s response. The use of the I/O

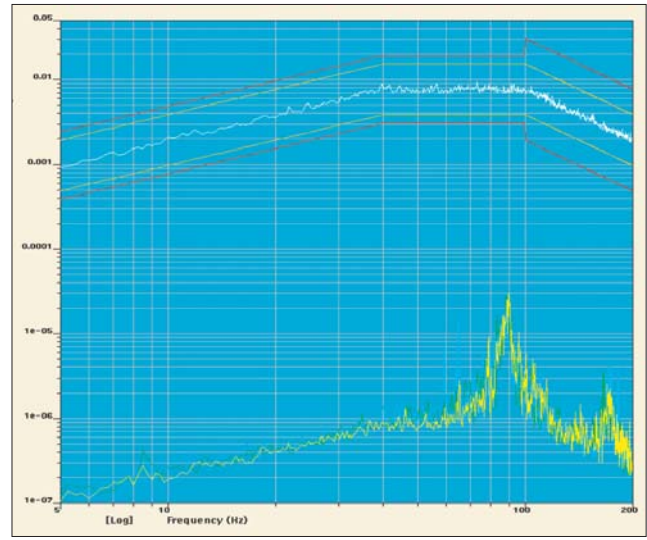


Figure 14. Vertical, roll, pitch, and torsion control vector components, 4-DOF control.

transformation matrix also reduces the problems of controlling a near singular SDM, as high coherence between measurement points creates.

Perhaps the most dramatic improvement offered by controlling torsional motion in a 4-DOF control scheme can be seen by comparing Figures 17, 13, and 6. The addition of a torsional control degree of freedom has assured that the coherence between control accelerometers never falls below 0.8 and that actual phase between accelerometers is within ±10° at virtually all frequencies. In fact, a coherence value of exactly 1.0 is achieved for a large portion of the control frequency range, something that is rarely possible with either 3-DOF or direct-actuator control.

### Observations

Based on the experiments described here, as well as previously reported results,<sup>(1, 2)</sup> some relevant observations can be made:

1. By forming a single, vertical, control vector from multiple transducers mounted in the same direction through a transformation matrix, a true average control paradigm is established.
2. The average control formed is independent of control transducer placement on the table or test article.
3. The “average control,” which sometimes occurs during rectangular control of multiple exciters, depends very much on control transducer placement. In fact control transducer placement during rectangular control testing may create a virtual “limit control” condition instead of average control, or a combination of both depending on transducer placement.
4. By eliminating unwanted degrees of freedom in the response of the table or structure, we are able to achieve much higher coherence between translational responses than through other more direct approaches.
5. By using the discussed I/O transformation matrix, problems associated with controlling what is otherwise a near singular SDM are significantly mitigated.

### Near-Singular Spectral Density Matrix (SDM)

As noted previously, the control that results in nearly pure translational motion will result in a nearly singular SDM. This will create numerical problems for the controller, which will cause control problems. For example, the coherence should not be specified as 1.0 between control points, when using square control. This is because the resulting SDM is singular and is difficult to realize in practical testing with square control due to the fact that this control depends on keeping differences between control responses near zero, while individual responses may be large. Rather, a value between 0.95 and 0.99

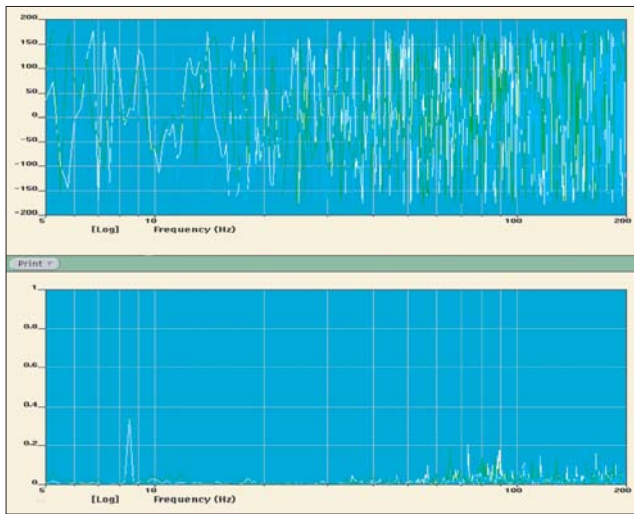


Figure 15. Control phase and coherence,  $g^2$  and  $f$  set to 0; 4-DOF control.

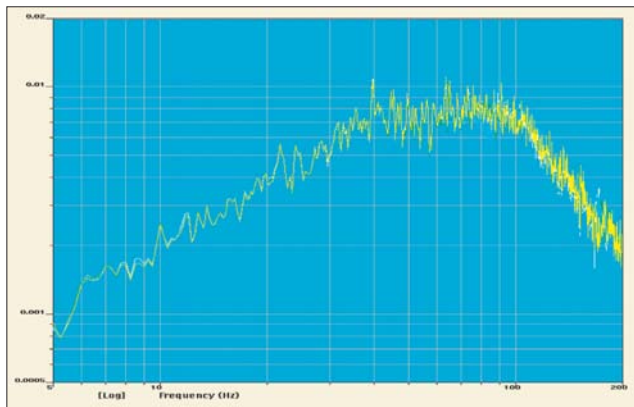


Figure 16. Response of four vertical accelerometers during 4-DOF control.

will typically yield excellent results without creating the potential numerical problem that specifying a relative coherence of 1.0 will create. However, as discussed earlier, the use of I/O transformations usually mitigates problems associated with attempting to create a singular SDM by indirectly causing the relative coherence of the four corners of the plate to reach values higher than 0.999. This subject is discussed in more detail in Reference 11.

## Conclusions

The transformation of signals from control transducers into control vectors is a very powerful technique. Although it has been used for some time in seismic testing to create a 6-DOF control from vertical, horizontal, and lateral actuators, most of these implementations were achieved using hardware summations and subtractions. Recent developments in control software have moved the ability to create flexible transformations of both signal inputs and drive outputs directly into real-time operating control systems. Incorporating this capability within the control system also avoids problems caused by different subsystems. Such problems include the use of multiple ADC and DAC conversions that inject additional quantizing errors in the respective drive and control response signals that the control system would need to use.

This newfound transformation flexibility has also created an opportunity to extend the concept to the control of tables, plates, and fixtures that may normally exhibit severe torsional responses within the test frequency range. By using transformation matrices derived from the physical system geometry and control transducer placement, a significant improvement in test control accuracy can be achieved. Beginning with translational control and successively adding control degrees of free-

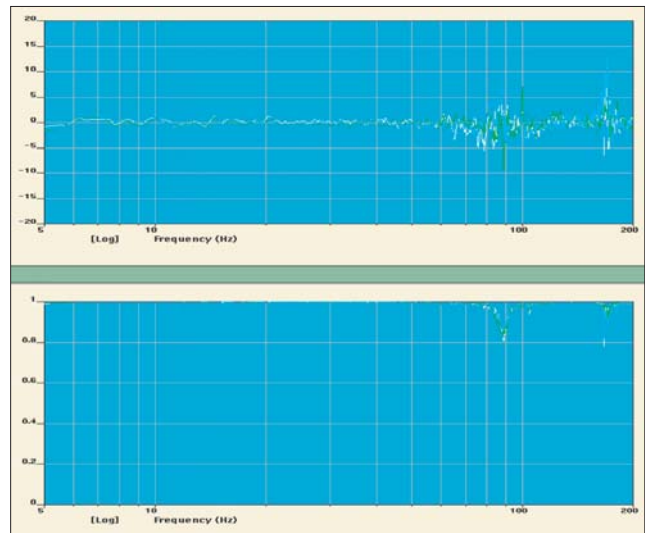


Figure 17. Phase and coherence between four vertical accelerometers for 4-DOF control.

dom, using the same control accelerometers, each new DOF adds to our ability to make a plate move like a uniform platform. Quite often, this is enough to make an entire test successful. Additionally, the use of I/O transformations has been shown to increase the relative coherence that is achieved between control accelerometers over what is possible with square control.

The renewal of interest in multiple-exciter testing and the added flexibility it offers in testing large structures can now be further enhanced by introducing additional testing degrees of freedom. This combination offers great promise in achieving test results never before thought possible and explaining structural behavior that may have gone unreported for many years. Much work remains to be done in making these benefits more apparent to the structural dynamics testing community.

## Acknowledgments

The authors wish to acknowledge the most able assistance of Russ Ayres and Paul Hawk in acquiring the test plate data and Dr. Jifang Tian in developing the transformation software used in these experiments.

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